COMMONWEALTH OF AUSTRALIA

Copyright Regulations 1969

Warning

CHARLES DARWIN UNIVERSITY

Family Name	
Given Names	
Student Number	
Teaching Period	Semester 1, 2016

FINAL EXAMINATION	**DURATION**
SMA101 – Mathematics 1a	Reading Time: **10** minutes Writing Time: **180** minutes

INSTRUCTIONS TO CANDIDATES

EXAM CONDITIONS

You may begin writing from the commencement of the examination session. The reading time indicated above is provided as a guide only.

This is a CLOSED BOOK examination

Any non-programmable calculator is permitted

No handwritten notes are permitted

No dictionaries are permitted

ADDITIONAL AUTHORISED MATERIALS	**EXAMINATION MATERIALS TO BE SUPPLIED**
No additional printed material is permitted	1 x 20 Page Book 1 x Scrap Paper Formula Sheet/s

THIS EXAMINATION IS PRINTED DOUBLE-SIDED.

THIS PAGE HAS BEEN INTENTIONALLY LEFT BLANK.

Question 1

(a) Find the solutions of the following inequalities and sketch them on the coordinate line:

(i) $\frac{3}{x} - 2 < 8$ and (ii) $3 \le 4 - 2x < 7$. **[Marks 6]**

(b) (i) Find the natural domain of the function $f(x) = \sqrt{x+3} + 5$ and express it in terms of closed and open intervals. **[Marks 3]**

(ii) Show the interval $(-\infty,-2] U [2,10)$ on the coordinate line. **[Marks 3]**

(c) Solve the following equalities:

(i) $x^2 - 4x > 12$ and (ii) $\frac{x}{x-3} < 4$. [**Marks 8]**

Question 2

(a) By translating the graph of $y = x^2$, sketch the graph of $y = x^2 - 6x + 13$.

[Marks 4]

(b) Find expressions of $f \circ g$ and $g \circ f$, and state the domains of each of the compositions.

Given that: $f(x) = \frac{1+x}{1-x}$ and $g(x) = \frac{x}{1-x^2}$. **[Marks 6]**

(c) Find the amplitude and period of: $y = 5 + \frac{2}{5}\cos\left(\frac{t}{3}\right)$. **[Marks 4]**

(d) Find the expression for $f^{-1}(x)$ if it exists for: $f(x) = \frac{x^2 - 1}{x^2 + 1}$. Sketch it and apply the horizontal line test and determine if $f^{-1}(x)$ is one-to-one? **[Marks 6]**

Question 3

(a) Find the following limits:

(i) $\lim_{x\to+\infty} \frac{\sqrt{5x^2-2}}{x+3}$ and (ii) $\lim_{x\to1^+} \frac{x^4-1}{x-1}$ **[Marks 5]**

(b) Find a value of the constant *k*, if any, that will make the following functions continuous:

$$f(x)=\begin{cases} kx^2, & x\le 2 \\ 2x+k, & x>2 \end{cases}$$ **[Marks 4]**

(c) A robot moves in the positive direction along a straight line so that after *t* minutes its distance is $s = 6t^4$ feet from the origin.

(i) Find the average velocity of the robot over the time interval [2,4] minutes.

(ii) Find the instantaneous velocity of the robot at $t = 2\,\text{min}$. **[Marks 6]**

(d) Find *dy*/*dx* of the following functions by implicit differentiation:

$x^2y+3xy^3-x=3$. **[Marks 5]**

Question 4

(a) Compute the derivative of the following function:

$$f(x) = (2x^6 - x^2)\left(\frac{x-1}{x+1}\right).$$ **[Marks 5]**

(b) A conical water tank with vertex down has a radius of 10 m at the top and is 24 m high. If water flows into the tank at a rate of 20 m^3/min, how fast is the depth of the water increasing when the water is 16 m deep? **[Marks 10]**

(c) Evaluate the following integral:

$$\int \frac{1}{1+\sin\theta} d\theta$$ **[Marks 5]**

Question 5

(a) Find a system of linear equations corresponding to the following augmented matrix.

$$\begin{bmatrix} 3 & 0 & -2 & 5 \\ 7 & 1 & 4 & -3 \\ 0 & -2 & 1 & 7 \end{bmatrix}$$ **[Marks 5]**

(b) Express the following system of linear equations in the matrix representation given by: $AX = B$, and then find matrices A, X and B.

$$\begin{aligned} 2x_1 - 3x_2 + 5x_3 &= 7 \\ 9x_1 - x_2 + x_3 &= -1 \\ x_1 + 5x_2 + 4x_3 &= 0 \end{aligned}$$. **[Marks 5]**

(c) Using the property of its determinant, show that the following matrix is not invertible:

$$\begin{bmatrix} \sin^2\alpha & \sin^2\beta & \sin^2\gamma \\ \cos^2\alpha & \cos^2\beta & \cos^2\gamma \\ 1 & 1 & 1 \end{bmatrix}$$ **[Marks 5]**

(d) Find the dot product **a.b** of the following vectors and express it in simplest form:

$$\mathbf{a} = (t, t^2),\ \mathbf{b} = (\cos^2 t, \frac{\sin^2 t}{t})\ .$$ **[Marks 5]**

Question 6

(a) For any two vectors **u** and **v**, show that:

$$\mathbf{u}.\mathbf{v} = \frac{1}{4}[\|\mathbf{u}+\mathbf{v}\|^2 - \|\mathbf{u}-\mathbf{v}\|^2] \ . \qquad \textbf{[Marks 5]}$$

(b) Find two unit vectors that are orthogonal to both **u** = (1, 0, 1) and **v** = (0, 1, 1).

[Marks 5]

(c) Express the following complex expression in the form of $a+bi$:

$$i(1+7i)-3i(4+2i) \ . \qquad \textbf{[Marks 5]}$$

(d) Express the following complex number in polar form:

$5 + 5i$. **[Marks 5]**